Stability and Strength Criteria for High Performance Steel Plates under Egde Compression

Güven Kıyımaż *

Abstract
A study of imperfect plate panels under compression is presented with emphasis being given on the effect on plate strength of different material and initial imperfection characteristics of high strength steel plates, in comparison to mild steel plates. Non-linear finite element analyses of imperfect plate panels were conducted over a practical range of plate slenderness. More realistic material behavior and residual stresses were adopted in modeling of high strength steel plate panels. Ultimate strength trends for mild steel and high strength steel plates over a wide range of plate slenderness were compared which led to a simple design equation that would account for the observed difference in non-dimensional plate strength of imperfect high strength steel plates.

Özet

Keywords: high performance steel, plate stability, plate imperfections, finite element analysis

1. Introduction
Modern innovative technologies in steel production have made it possible to manufacture steels with yield strengths in excess of 690 MPa, so called High Performance Steels (HPS), and with low yield-to-ultimate strength ratio, while favorable weldability is maintained. Improved mechanical and weldability properties of high strength steels have increased the scope for structural applications.

Thin walled box section columns fabricated from high yield strength steel plates is one of the areas of interest. However, as with every new material that becomes available, lack of code / design guidance is an important barrier that prevents the efficient use of high performance steel plates in structural members. Steels with the above mentioned high performance mechanical and weldability properties are currently available only in plates and the present European code provisions cover steel grades with nominal yield strength values of up to 460 MPa and are only applicable to rolled I-sections.

* Department of Civil Engineering, İstanbul Kültür University, 34 191 Şirinevler / İstanbul
In view of this lack of design guidance, the paper attempts to provide the necessary scientific basis on ultimate strength of plate elements of high performance steels used as components of fabricated box section columns under axial compression. The study involves a parametric non-linear finite element analysis of imperfect plates over a range of plate slenderness values. Plates with different steel grades and with relevant initial imperfections were considered. Particular emphasis was given on modeling high performance steel plates with more realistic material behavior and residual stresses. Non-dimensional stress-strain curves are presented for plates with different steel grades and with distinct plate slenderness values namely stocky, intermediate and slender. Non-dimensional ultimate strengths of uni-axially compressed imperfect plates are presented for different steel grades over a wide range of plate slenderness. Results are compared against available design formulations. A notable increase in non-dimensional plate strength for high performance steel plates with high plate slenderness values is reflected in the design formulations with an additional term.

2. Finite Element Analyses Of Uni-Axially Compressed Plates
2.1 Geometry and Material

Figure 1 shows the plate that was analyzed on ABAQUS (1995). The numerical model of the plate was developed on the basis of the following assumptions;

- Square geometry to represent a single buckle of a long plate element.
- Simply supported along the four edges and constrained to move uniformly in a direction perpendicular to the edges so that the expected mode of local buckling in a box section could be achieved.
- The thickness of the plate was varied giving the flexibility to carry out the analysis for a range of width-to-thickness ratios.
- A linear multi-point constraint equation was imposed on both loaded edges of the plate. Hence the load was applied as uniform displacement.
- For the eigenvalue buckling analysis, the whole plate was modelled in order to compare anti-symmetric modes, however, in the non-linear analysis only one quarter of the plate was analyzed and hence symmetry boundary conditions were adopted.

General shell stress / displacement elements (quadrilateral doubly curved thin shell with 9 nodes using five degrees of freedom per node) were used for the finite element analyses. This type of element (S9R5) is intended for thin shell applications and thus suitable for capturing the local buckling modes. The mesh adopted is a result of the equilibrium requirements for the longitudinal residual stress state for various yield strength plates, i.e. the dimensions of the finite elements are sufficient for modeling different residual stress state conditions. This is discussed in detail in 2.4.

For the eigenvalue buckling prediction, material is assumed to be isotropic elastic, with a Young’s modulus of 210000 MPa and Poisson’s ratio 0.3. For further load-displacement analysis, plasticity is also included in the material definition. For high strength steel this implies the use of a Ramberg-Osgood model, whereas for mild steel an elastic-perfectly plastic model is used.
2.2 Analysis procedure

Two types of analysis are carried out to study plate stability. First, eigenvalue buckling analysis is carried out to obtain estimates of the critical buckling stresses and corresponding buckling modes. This analysis provides guidance in terms of assessing the reliability of the mesh, load and boundary conditions assumed for the model. Secondly, non-linear analysis is carried out using the mesh and imperfections suggested by the eigenvalue analysis. Riks method is used for the non-linear load-displacement analysis to handle possible instabilities that the plate would suffer due to the presence of initial geometric and material imperfections.

2.3 Eigenvalue buckling analysis

Eigenvalue analysis was basically carried out for validation purposes, i.e. to check whether the Finite Element package performs as anticipated.

Eigenvalues, i.e. the critical buckling loads, were found for the first three modes of instability. The critical loads or stresses have been compared against the theoretical values and are given in Table 1. As can be seen, they were found to be in very close agreement with the theoretical values. A 12 x 12 mesh was adopted for the whole plate. Using coarser meshes, such as 2 x 2 or 4 x 4 did not give satisfactory results.

Critical buckling stresses depend on the initial stiffness of the plate. The aforementioned results were for a perfect plate. If in-plane residual stresses are present in a perfect plate before the load is applied, the initial stiffness of the plate changes and this modifies the critical stress value. Results of the eigenvalue analysis of the plate with residual stresses (tension on a narrow strip at the edges and compressive at the middle region), as in Figure 2, supported the above argument, giving lower values for the critical buckling stresses. These results are given in Table 2. It is noted that the same mesh (12 x 12) was adopted for the eigenvalue analysis of the plate with residual stresses. For the assumed compressive residual stress-to-yield stress (f_y) ratios for different yield strength values given in Table 2, the width of the elements is determined so that an equilibrium between tensile residual stress at the edges and compressive residual stresses in the plate centre is achieved. Figure 3 shows the first three buckling modes.

2.4 Non-linear analysis

Following the eigenvalue analysis, non-linear analysis was carried out. The main purpose is to investigate (numerically) load-deformation behaviour and the ultimate strength of steel plates assuming different yield strengths, with specific initial geometric imperfections and welding residual stresses.

A validation study was first carried out which comprised of a set of non-linear finite element analysis of plate panels with various geometric imperfections. The parameters, i.e. imperfection amplitudes and the plate slenderness values, used for this set of analyses have been taken from Dubas and Gehri (1986) and results obtained from ABAQUS (1995) are compared with their results. The comparison is presented in Figure 4, and is considered very good. For \( \lambda_p = 1.1 \), which has been adopted in this set of analyses, as the imperfection gets smaller unloading of the plate in the post-ultimate region becomes sharper, whereas it becomes smoother for larger imperfections. Note that for the results presented in Figure 4, no residual stresses have been considered.
2.4.1 Residual Stresses

One important aspect of welded HPS plates is the fact that they are believed to exhibit lower residual stress to yield stress ratios compared to mild steel plates. Thus, a study has been carried out to determine the practical range of residual stresses that would be present in welded steel plates in normal and high strength steel plates. The values that have been estimated based on analytical (semi-empirical) models were then compared with experimental results presented by other researchers and finally approximate distributions of residual stresses for different yield strengths were adopted and introduced in the Finite Element Analysis.

2.4.2 Residual Stress Distribution

The welding of steel plates gives rise to very high temperatures, which result in considerable residual stresses after cooling. Beside the welds, the metal and the welds are stressed in tension normally up to yield. This region carries locked-in tension stresses while the rest of the section is in a state of residual compression for equilibrium.

Dwight’s model (1969) has been adopted by many investigators for the residual stress distribution in edge welded plates (This was shown schematically in Figure 2). This idealized residual stress pattern is based on experimental results on welded box-section columns and is convenient for analysis. The central compressive stress region is assumed to be in balance with the tension regions of width $ht$ at the edges. Longitudinal stress equilibrium leads to the following equation:

$$\sigma_{rc} = \frac{2\eta}{b/t - 2\eta} \sigma_{rl}$$

(1)

2.4.3 Maximum Tensile Residual Stresses

Figure 5 presents a plot showing the maximum welding residual stresses, which occur as locked-in tension in the vicinity of the weld, as a function of various yield strength values. These curves are based on the experimental results obtained by Ikeda and Kihara (1970) and Akita and Yada (1965), as reported in Masubuchi (1980). These researchers studied residual stresses in butt welded plates of various base metal yield strengths. The results clearly show that the rate of increase of the maximum tensile residual stress present beside the weld is decreasing with increasing yield strength.

Hwang (1976) conducted an analytical and experimental study on the thermal stress and metal movement that occurs during welding. In the experiments two types of weld were tested; weld bead being on the edge of plate and butt weld. Plate material included low carbon steel, Quenched & Tempered steel (A517) and HY-80 steel, where the last two (A517 and HY-80) are high strength steels. Residual stress distributions in weldments of these three different steels were presented for a particular weld type. In the weldments of low-carbon steel, the experimental data and the analytical predictions agreed but in the weldments of A517 and HY-80 steel the test results in regions beside the weld were significantly lower than the analytical predictions. Figure 6 shows Hwang’s experimental and theoretical results for the distribution of residual stresses in
the Quenched and Tempered steel (ASTM A517) plate with a base metal yield strength of about 690 MPa. The weld was on the edge of the plate and welding was carried out by gas metal-arc process.

From Figure 5, for 690 MPa base metal yield strength steel, the average maximum tensile residual stress (beside the weld) is estimated as 415 MPa. Again from this curve average maximum tensile residual stresses for 275 MPa and 460 MPa base metal yield strength steels are approximately equal to 275 MPa and 390 MPa respectively.

2.4.4 Compressive Residual Stress and Extent of Tensile Zone

Assuming the maximum tensile residual stresses are approximately as derived above, an attempt to calculate the equilibrating compressive residual stresses in two different ways is presented below.

The equilibrium equation for the Dwight model residual stress distribution is composed of three parameters of which two are independent. If \( \sigma_n \) is assumed to be determined from experimental results, an estimate for either \( \sigma_n \) or \( \eta_t \) should be made in order to solve the equilibrium equation (1). The two methods presented below include estimating \( \eta_t \) in different ways and finding \( \sigma_{rc} \) from equilibrium.

- Bonello’s Statistical Model

Bonello (1992) has come up with a model for estimating the extent of the tensile residual stress zone adjacent to the weld through regression analysis of available test results. As a result of this analysis, the following formulas have been suggested

\[
\eta = \eta_R + \Delta \eta
\]  

(2)

where \( \eta \) is the width per unit plate thickness of the tensile yield block, \( \eta_R \) is the mean value of \( \eta \) given as;

\[
\eta_R = 1.40 + 0.06 \frac{b}{t}
\]  

(3)

and \( \Delta \eta \) is a normal zero-mean random variable with a standard deviation obtained from regression analysis of test results as;

\[
\sigma_{\Delta \eta} = 0.04 \frac{b}{t}
\]  

(3)

Table 3 shows the results for compressive residual stress values as a percentage of three base metal yield strengths using Bonello’s approach (1992). The calculations have been done for a particular plate reference slenderness \( (\lambda_p = 1) \) and a width of plate \( (b=720 \text{mm}) \) thus for a range of thickness \( (t) \) values. The relation between \( \lambda_p \) and \( t \) for a particular yield strength \( f_y \) is given as \( \lambda_p = \sqrt{\frac{f_y}{\sigma_{ct}}} \). Two values of \( \sigma_{rc} \) have been calculated; for \( \eta_1 = \eta_R \) and \( \eta_2 = \eta + \sigma_{\Delta \eta} \) as average and upper bound estimates.
• *Shrinkage Force Approach*

According to Dwight (1969), for a plate of given thickness and with a given thermal history at its edges, i.e. a particular weld, the extent of the tension residual stress region, \( \eta_t \), is mostly independent of the overall width \( b \) (provided that \( b/t \) is over a certain limit; as the plate gets more slender, i.e. high \( b/t \), the level of \( \sigma_{tc} \) drops due to the increase in the extent of middle compressive region without changing the extent of the tension regions). Therefore it is the weld size and properties which primarily affect \( \eta_t \).

The following expression has been derived by Dwight (1969) for the compressive residual stress, \( \sigma_{tc} \):

\[
\sigma_{tc} = \frac{\sigma_{tt}}{\sigma_{tc} (b/t)} - 1
\]

(5)

where \( C \) is a constant which depends on the type of welding. Available experimental data on residual stresses suggest \( C = 6000 \) MPa as a reasonable figure for practical use (Dwight 1969). Other parameters of equation (5) are shown in Figure 7.

In (5), \( \sigma_{tc} \) is proportional to \( w/t \). This means that for a given plate thickness, higher residual stresses are expected as the weld size increases. Equation (5) is based on the following two equations; (1), which describes the equilibrium of the residual stress distribution and (6) below which gives an expression for \( \eta_t \) as a function of the weld properties.

\[
\eta_t = \frac{0.6w^2C}{\sigma_{tc} 2t}
\]

(6)

The extent of the tensile residual stress region in (6), as explained in Dwight (1969), is derived first as a function of welding parameters such as current, voltage, speed and process type. Noting that the metal added in the weld is nearly equal to the amount of electrode metal and relating this amount with the aforementioned welding parameters, equation (6) is expressed as a function of the weld size which is more suitable for designers.

Table 4 shows three different values of compressive residual stresses for three different base metal yield strengths, this time using the Shrinkage Force Approach and a value for \( C = 6000 \) MPa as mentioned above. Also presented in this table are the compressive residual stresses which are calculated for a value of \( C = 10400 \) MPa which is based on Vinokurov’s calculations (1968).

### 2.4.5 Final Residual Stress Distribution

Usami & Fukumoto (1982) made residual stress measurements on 690 MPa steel plates of varying \( b/t \) ratios by the sectioning method. Tensile residual stresses of about 60% of yield strength were measured beside the weld. This value for \( \sigma_{rt} \) closely matches the theoretical residual stress calculations reported in Masubuchi (1980). For three different \( b/t \) ratios, 22, 33 and 44, the average values of measured compressive residual
stresses were 14%, 9% and 11% of the yield strength respectively. Also in Nishino et. al. (1967) results of experimental residual stress measurements for A514 steel with static yield strength of about 700 MPa are reported. Compressive residual stress values of between 10% and 15% of base metal yield strength were measured.

According to the study discussed in the previous sections and the comparison made above with the experimental results for high strength steel, Table 5 presents the final adopted residual stress distributions for the three different base metal yield strengths, for a plate with slenderness \( \lambda_p = 1 \).

2.4.6 Geometric Imperfections

Unavoidable initial out-of-flatness can have a significant effect on the stiffness and strength of thin plates in compression. Because of the fact that such geometrical imperfections are not easily measured, predicting the behaviour of a real plate through numerical procedures becomes difficult. However, in design, the imperfections may be assumed to be kept within certain tolerance limits and the uncertainty regarding their effect is reduced. In this study, following the philosophy of many earlier studies, conservative assumptions are made regarding the modelling of geometric imperfections.

For this purpose, the following expression for allowable geometric out-of-flatness (tolerance) given in BS5400 (1980) is used and this out-of-flatness amplitude is assumed to be affine to a half-sine wave mode (i.e. the critical buckling mode).

\[
\Delta w = \frac{G}{165} \left( \frac{f_s}{355} \right) \quad \text{or 3mm, whichever is greater}
\]  

(7)

The particular geometry adopted here (b=720mm and thicknesses for three yield strength values) is chosen such that imperfection tolerance values for all three cases, i.e. three yield strengths, are greater than the lower limit of 3mm. In this way, a better comparison between different steel grades can be achieved, since the imperfection amplitude is a function of material yield strength. Table 6 presents the maximum plate imperfection amplitudes adopted for each yield strength value.

The value reported in Table 6 correspond to \( Dw/t = 0.28 \) for \( \lambda_p = 1 \).

2.4.7 Material Model for 690 MPa Steel

Quenched & Tempered high strength steel does not exhibit a well defined yield plateau. In contrast to mild steel, which exhibits a bi-linear stress-strain relationship with a well-defined yield plateau, the stress-stain material characteristics of high performance steels are often modeled through a Ramberg - Osgood expression.

In this study, an approximate material model as shown in Figure 8 has been adopted. This approximation has been made by digitizing the stress-strain diagram obtained from tensile coupon tests of Fukumoto & Usami (1982). These tests were carried out on specimens with nominal yield stress of about 690 MPa (HT80 steel which is equivalent to ASTM 514 steel). The Ramberg - Osgood expression is given by:

\[
\varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_{0.2}} \right)^n
\]

(8)
To define a particular point on the stress-strain curve three independent constants are required; E, $\sigma_{0.2}$ and n. Here n is an index which defines the degree of non-linearity of the curve and can be worked out by the following formula;

$$n = \log(0.5) / \log(\sigma_{0.1}/\sigma_{0.2})$$

(9)

Again from the stress-strain curve presented by Fukumoto & Usami (1982) the index n is calculated as 5.3 with $\sigma_{0.2} = 690$ MPa and $\sigma_{1.0} = 740$ MPa. The elastic modulus, E, is taken equal to 210000 MPa.

### 2.5 Numerical parametric study

Considering all items mentioned in the previous sections, various analyses have been carried out using ABAQUS for plates with base metal yield strength 275 MPa, 460 MPa and 690 MPa. A discussion of the results is presented below.

Figures 9, 10 and 11 illustrate the response of a plate with $\lambda_p = 1$ in compression with yield stress equal to 275 MPa, 460 MPa and 690 MPa respectively, with geometric imperfections and residual stress distributions as determined in the preceding section. Four different cases have been analyzed to observe separately, and in combination, the effect of manufacturing distortions on plate response; plate with no geometrical imperfections or residual stresses, plate with residual stresses only, plate with geometrical imperfections only and plate with both imperfection and residual stresses.

If the plate contains no distortions, then the behaviour of a plate with $\lambda_p = 1$ is similar to the material response. For example, for 275 MPa steel plate with no distortions the load-end shortening curve follows the elastic-perfectly plastic material model, whereas for 690 MPa steel plate without distortions, the corresponding curve follows the Ramberg- Osgood material model.

The presence of residual stresses has no significant effect on plate behaviour. Initially, however, once yielding begins due to the combination of the applied stress and initial residual stress, both stiffness and strength are reduced. At higher strains, stresses become equivalent to that of the initially stress free plate (material response) so that the effect of residual stresses practically disappear and both responses become more or less identical. For 275 MPa and 460 MPa steel plates, it is observed that the reduction in strength due to residual stresses is significant when yield strain, $\varepsilon_y$, is reached and stress-strain curve converges to that of the material response at about twice the yield strain ($2\varepsilon_y$).

Stress-strain curve for 690 MPa steel with residual stresses is similar to the material response. There is no apparent reduction in strength at the yield strain level and the aforementioned convergence of the stress-strain curve to the material response curve appears to occur more or less at the yield strain level.

Introducing a geometrical imperfection in the form of an initial bow effects the stiffness and strength of the plates from the onset of loading and this continues over the complete strain range. Effect of geometrical imperfections on ultimate strength is much more significant than the effect of residual stresses. Compared with the ultimate strengths for perfect plate panels, percentage reduction in strength due to presence of
only geometrical imperfections ranges from 32% to 38%. Comparing the ultimate strengths achieved for the "Only geometrical imperfections" and "Both imperfections" cases, it was calculated that the percentage ultimate strength reduction ranges from 1% to 6% for three values of yield stresses.

Figures 12 through to 17 illustrate the effect of yield strength on the response of uni-axially compressed imperfect plates for three different plate slenderness values; first a plate slenderness in the stocky region ($\tilde{\lambda}_p=2$), secondly a plate in the intermediate region ($\tilde{\lambda}_p=1$) and thirdly a plate in the slender region ($\tilde{\lambda}_p=2$). Geometric imperfections and residual stresses have been adopted as explained above. Main purpose of these curves is to compare plate ultimate strength for various yield strength values and plate slendernesses on a non-dimensional basis. Figures 12, 14 and 16 are for cases where only the geometric imperfections are present and Figures 13, 15 and 17 are for both geometrical imperfections and residual stresses present. The objective of presenting the stress-strain curves in this way is to reveal the effect of residual stresses on ultimate strength of plates with varying residual stress levels.

On a non-dimensional basis, geometric imperfection values adopted using the BS 5400 (1980) imperfection tolerance as explained above and given in Table 6, affects equally the ultimate strength for intermediate and slender plate slenderness. For stocky plates, the behaviour is similar to the previously mentioned material response. It is also noted here that increasing the plate slenderness reduces the post-ultimate stiffness of the plate.

The effect of the proportionately lower $\sigma_{tc}$ / $f_y$ ratio for higher strength steel is notable especially for the slender plate. Thus, the strength gains for 690 MPa steel compared to mild steel are in the range of 4% to 12%.

2.6 Comparison Of FE Results With Design Formulations

In Figure 18, non-dimensional ultimate strengths of uni-axially compressed imperfect plates (with geometrical imperfections and residual stresses) are presented for the aforementioned yield strengths over a range of plate slenderness. Also presented in Figure 18 are the elastic buckling curve, Von Karman’s post-buckling curve, Eurocode 3 Part 1.1 (1993) and BS 5400 (1980) plate strength curves, as well as the curve proposed by Chou (1997). The influence of yield strength is small for intermediate values of plate slenderness while it is gradually increasing with increasing plate slenderness. Also in the stocky range, high strength steel plates seem to be stronger than normal grade steel plates when compared on a non-dimensional basis.

The conclusion regarding the increase in the difference in non-dimensional strength of mild and high strength steel plates for higher plate slenderness values was also drawn by Rasmussen & Hancock (1992) and by Nishino et al. (1967). The difference in ultimate strength is up to 12% for a plate slenderness value of ($\tilde{\lambda}_p=2$). As use of High Performance Steel is likely to lead to more slender construction, such a difference can be an important factor in material selection for weight critical applications.

The difference between ultimate strengths of mild steel and HPS plates is mainly attributable to the different residual stress levels adopted, which is mainly reflected in
the high plate slenderness region. In the stocky plate region, the higher non-dimensional ultimate strength for HPS plates may additionally be attributed to strain hardening characteristics of the Ramberg-Osgood model.

The strength of the aforementioned plates, which were predicted by FE, were also calculated using a recently developed semi-empirical formula. A set of new effective-width formulae has been derived as functions of both initial plate out-of-flatness and residual stresses (Usami 1993). These formulae are based on an extensive numerical study on elasto-plastic large-displacement finite element analysis of simply supported initially imperfect plates under uni-axial compression. These plate strength equations, as mentioned in Usami (1993), can be applied to compute the strength variations of stiffened plates and welded box sections by explicitly including the effects of both residual stresses and initial out-of-flatness. Maximum strength, \( \sigma_m \), of a uni-axially compressed plate with initial distortions is given as follows:

\[
\frac{\sigma_m}{\sigma_y} = \frac{1}{2}\frac{1}{\lambda_p} \left[ \beta - \sqrt{\beta^2 - 4\lambda_p} \right]
\]  

in which

\[
\beta = 1 + C.(\lambda_p - \lambda_{n0}) + \lambda_p
\]  

\[
\lambda_{n0} = A - B.\ln\left(\frac{\Delta_p}{b}\right) \leq 1.0
\]  

\[
A = -0.05 - 0.542.\exp\left(-11.9\frac{\sigma_{rc}}{\sigma_y}\right)
\]  

\[
B = -0.09 + 0.107.\exp\left(-12.4\frac{\sigma_{rc}}{\sigma_y}\right)
\]  

\[
C = -157\left(\frac{\Delta_p}{b}\right)\left(\frac{\sigma_{rc}}{\sigma_y}\right) + 43\left(\frac{\Delta_p}{b}\right) + 1.2\left(\frac{\sigma_{rc}}{\sigma_y}\right) + 0.03
\]  

where \( \lambda_p \) is the plate reference slenderness given as;

\[
\lambda_p = \frac{f_y}{\sigma_m} \left[ \frac{b}{f} \right] \left[ \frac{12(1-\nu^2)}{\pi^2k} \right] \frac{f_y}{k}
\]  

and \( \Delta_p \) is level of initial out-of-flatness, \( \sigma_{rc} \) is the level of compressive residual stresses, \( b \) is plate width, \( t \) is plate thickness and \( f_y \) is the yield strength of the plate material.
FE plate strength estimates, given earlier on non-dimensional strength-plate slenderness plots, are compared with plate strengths given by the above strength equations which explicitly take into account the initial geometric imperfections and residual stresses. The comparisons are made both for mild steel and HPS plates and are given in Figures 19 and 20. It appears from these figures that, for mild steel plates, there is good agreement between ABAQUS and the Usami plate strength estimates. On the other hand, for HPS steel plates, ABAQUS strength predictions are higher than those due to Usami. Usami’s plate strength equations are based on FE results generated for plates with yield strength of 350 MPa. The discrepancy for the HPS plates (690 MPa) may therefore be because the strength equations have not been calibrated for high yield strength plate cases. It is also worth noting that both Usami’s estimates and the current ABAQUS results fall below the EC3 predictions in the intermediate plate slenderness range (around $\lambda_p = 1$).

Figure 21 compares plate strengths for mild steel and HPS, both calculated using Usami’s strength equations. Assuming the same residual stresses and initial plate out-of-flatness values that were used for the ABAQUS plate analyses, Usami’s equations provide a similar trend on a non-dimensional basis to that obtained by ABAQUS, i.e. higher non-dimensional strengths for HPS. It would thus be feasible to calibrate A, B and C for HPS predictions, which should take into account the different material model applicable to this grade of steel.

A simple way of introducing the increase in strength (due to proportionately lower residual stress levels) is to include an additional term in design strength formulations. For example, a third term may be added to the EC3 plate formulations as follows:

\[
\overline{\lambda}_p \leq 0.673 \quad \rho = 1
\]

\[
\overline{\lambda}_p \leq 0.673 \quad \rho = \frac{1}{\overline{\lambda}_p} - \frac{0.22}{\overline{\lambda}_p^2} + 0.025 \cdot \overline{\lambda}_p,
\]

\[
\rho \text{ smaller than } 1.
\]

(17)

Since $\overline{\lambda}_p$ is a function of $\varepsilon = \sqrt[3]{235/f_y}$ such a term would account for steel grades higher than the normal. Equation (17) is shown for HPS plates in Figure 20.

2.7 Concluding Remarks

In this paper, the effect of steel grade on uni-axial plate strength was studied. Plates were assumed to have initial geometrical imperfections and residual stresses due to welding. Following the eigenvalue analyses of uni-axially compressed square plate panels, non-linear analyses were carried out in a parametric study.

Emphasis was given on modelling HPS plates with more realistic material behaviour and residual stresses. A procedure which involves estimating the residual stress levels by averaging the predictions from a statistical and a semi-empirical
approach was presented. This procedure was then used to model the residual stresses for welded plates with different material yield strengths.

Non-linear finite element analyses of imperfect plate panels were conducted over the practical range of plate slenderness. Results showed that HPS plates with distortions exhibit somewhat different characteristics compared to mild steel plates. On an ultimate strength versus plate slenderness relationship, the effect of more realistic residual stress levels in HPS plates was observed to be more significant for high plate slenderness values; higher non-dimensional ultimate strengths were achieved for HPS plates compared with those of mild steel plates.

The semi-empirical plate strength equations by Usami were used to compare against FE plate ultimate strength predictions both for mild steel and HPS. Good agreement was achieved. Usami’s equations, which are expressed as functions of initial imperfections and residual stresses, were useful to compare the difference between ultimate strength trends for mild steel and HPS plates over a wide range of plate slenderness. Using these equations, higher non-dimensional ultimate strengths were achieved for HPS plates with high slenderness. Similar results were found through FE. Thus, the trends of FE results were confirmed by Usami’s semi-empirical equations.

A simple proposal to account for the increase in non-dimensional plate strength for HPS plates was made by adding an extra term to the existing Eurocode 3 formula.

References
Table 1 Comparison of the theoretical and numerical critical buckling stresses, \(E = 210000\text{MPa}, \nu = 0.3\)

<table>
<thead>
<tr>
<th>b /t</th>
<th>Theoretical (MPa)</th>
<th>Numerical (MPa)</th>
<th>Discrepancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.55</td>
<td>275</td>
<td>278.35</td>
<td>1.2</td>
</tr>
<tr>
<td>40.68</td>
<td>460</td>
<td>457.3</td>
<td>0.6</td>
</tr>
<tr>
<td>33.18</td>
<td>690</td>
<td>679.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Note:** Plate dimensions were chosen such that plate reference slenderness \(\lambda_p\) is 1, so that theoretical critical buckling stresses are equal to yield stress values of 275 MPa, 460 MPa and 690 MPa. Although yield stresses are not relevant in an eigenvalue buckling analysis, this is noted here to point out how the above b/t values were calculated.

Table 2 Effect of Residual Stresses on the buckling stress. 20\%, 17\% and 12\% compressive residual stress assumed for 275MPa, 460MPa and 690MPa steels respectively.

<table>
<thead>
<tr>
<th>Nominal fy (MPa)</th>
<th>Buckling modes</th>
<th>No residual stress (MPa)</th>
<th>With residual stress (MPa)</th>
<th>Reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>1</td>
<td>278.35</td>
<td>225.8</td>
<td>18.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>433.7</td>
<td>381.1</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>767.9</td>
<td>715.3</td>
<td>6.8</td>
</tr>
<tr>
<td>460</td>
<td>1</td>
<td>457.3</td>
<td>383.0</td>
<td>16.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>711.1</td>
<td>636.6</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1254.9</td>
<td>1180.3</td>
<td>5.9</td>
</tr>
<tr>
<td>690</td>
<td>1</td>
<td>679.9</td>
<td>600.6</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1054.6</td>
<td>973.0</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1853.6</td>
<td>1774.1</td>
<td>4.3</td>
</tr>
</tbody>
</table>

**Note:** A nominal fy is quoted so that the residual stress block can be estimated from the percentages given above, the idealized model shown in Figure 2.
Table 3 Compressive residual stresses following Bonello’s statistical model (1992), Note slenderness $\bar{\lambda}_p = 1$

<table>
<thead>
<tr>
<th>Nominal $f_y$ (MPa)</th>
<th>Plate thickness (mm)</th>
<th>Bonello’s Predictions $\eta$</th>
<th>$\sigma_{rc}/f_y$ for $\eta_1$</th>
<th>$\sigma_{rc}/f_y$ for $\eta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>13.7</td>
<td>4.55, 6.65</td>
<td>21%, 34%</td>
<td></td>
</tr>
<tr>
<td>460</td>
<td>17.7</td>
<td>3.84, 5.46</td>
<td>19%, 30%</td>
<td></td>
</tr>
<tr>
<td>690</td>
<td>21.7</td>
<td>3.39, 4.72</td>
<td>15%, 23%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Compressive residual stresses following Dwight’s shrinkage force approach (plate slenderness $\bar{\lambda}_p = 1$)

<table>
<thead>
<tr>
<th>Nominal $f_y$ (MPa)</th>
<th>Thickness (mm)</th>
<th>$\sigma_{rc}$ (MPa)</th>
<th>$\sigma_{rc}/f_y$ C = 6000 MPa</th>
<th>$\sigma_{rc}/f_y$ C = 10400 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>13.7</td>
<td>40.7</td>
<td>15%</td>
<td>19%</td>
</tr>
<tr>
<td>460</td>
<td>17.7</td>
<td>51.8</td>
<td>11%</td>
<td>14%</td>
</tr>
<tr>
<td>690</td>
<td>21.7</td>
<td>64.8</td>
<td>9.4%</td>
<td>12%</td>
</tr>
</tbody>
</table>

Table 5 Final compressive and tensile residual stresses for three different yield strength steels ($\bar{\lambda}_p = 1$)

<table>
<thead>
<tr>
<th>$f_y$ (MPa)</th>
<th>$\sigma_{rt}$ (MPa)</th>
<th>$\sigma_{rt}/f_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>275</td>
<td>20%</td>
</tr>
<tr>
<td>460</td>
<td>390</td>
<td>17%</td>
</tr>
<tr>
<td>690</td>
<td>415</td>
<td>12%</td>
</tr>
</tbody>
</table>

Table 6 Maximum allowable geometric out-of-flatness amplitudes for plates of various yield strengths ($\bar{\lambda}_p = 1$)

<table>
<thead>
<tr>
<th>$f_y$ (MPa)</th>
<th>Width or G as in BS5400</th>
<th>Thickness (mm)</th>
<th>$\Delta w$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>720</td>
<td>13.7</td>
<td>3.84</td>
</tr>
<tr>
<td>460</td>
<td>720</td>
<td>17.7</td>
<td>4.97</td>
</tr>
<tr>
<td>690</td>
<td>720</td>
<td>21.7</td>
<td>6.10</td>
</tr>
</tbody>
</table>
**Figure 1** Finite element model of the plate and applied load

**Figure 2** Idealized residual stress distribution
Figure 3  First three buckling modes for the uni-axially compressed plate
Figure 4  Effect of geometric imperfections on the behaviour of plates in compression

Figure 5  Maximum tensile residual stresses for various yield strength base plates (from Masubuchi 1980)
Figure 6  Distribution of residual stresses in the Quenched and Tempered steel. ASTM A517, Weld on edge of plate (from Masubuchi 1980)

Figure 7  Typical geometry for fillet welded plates

Figure 8  Material model for High Performance Steel.
Figure 9  Stress-strain behaviour of the uni-axially compressed plate for various imperfection cases, $f_y=275$ MPa, $\bar{\lambda}_p=1$

Figure 10  Stress-strain behaviour of the uni-axially compressed plate for various imperfection cases, $f_y=460$ MPa, $\bar{\lambda}_p=1$
Figure 11  Stress-strain behaviour of the uni-axially compressed plate for various imperfection cases, $f_y=690$ MPa, $\bar{\lambda}_p = 1$

Figure 12  Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Only geometrical imperfections present, $\bar{\lambda}_p = 0.2$
Figure 13  Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Both imperfections present, $\bar{\lambda}_p = 0.2$

Figure 14  Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Only geometrical imperfections present, $\bar{\lambda}_p = 1$
Figure 15 Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Both imperfections present, $\bar{\lambda}_p = 1$

Figure 16 Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Only geometrical imperfections present, $\bar{\lambda}_p = 2$
Figure 17  Non-dimensional stress-strain behaviour of uni-axially compressed plates with various yield strength values; Both imperfections present, $\bar{\lambda}_p = 2$

Figure 18  Non-dimensional strength of uni-axially compressed imperfect plates with various yield strength values over a range of plate slenderness.
**Figure 19** Plate ultimate strength estimates of ABAQUS compared against those of EC3 and those due to Usami (1993); Mild steel plates

**Figure 20** Plate ultimate strength estimates of ABAQUS compared against those of EC3 and those due to Usami (1993); HPS plates
Figure 21 Plate ultimate strengths due to Usami (1993); Mild steel plate cases vs. HPS plate