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THE EFFECT OF NOISE AND PARAMETER MISMATCH ON THE SYNCHRONIZATION OF THE DRIVE-RESPONSE CHAOTIC SYSTEM

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Abstract

We present a thorough investigation of the effect of noise (internal or external) on the synchronization of a drive-response configuration system (unidirectional coupling between two identical systems). Moreover, since in every practical implementation of a communication system, the transmitter and receiver circuits (although identical) operate under slightly different conditions it is essential to consider the case of the mismatch between the parameters of the transmitter and the receiver. In our work we consider the non-autonomous 2[nd] order non-linear oscillator system presented by G. Mycolaitis, *et al* in Proc. of 7[th] Int. Workshop on Non-linear Dynamics of Electronic Systems, which is particularly suitable for digital communications.

Index Terms *Chaos, nonlinear circuits, synchronization, communication system security.*

INTRODUCTION

Computer networks are inherently insecure for communication. Data transmission is not safe unless it is assured that the packets will never pass through a router or a computer, over which there is no control. Traditionally, software techniques were used for data encoding. However, the ever-increasing computer power threatens communication security.

The simplicity of chaos generators, the rich structure of chaotic signals and the fact that chaotic signals can be synchronized caused a significant interest in possible utilization of chaos for secure communications [1]-[3].

The use of synchronized chaotic systems for communications usually relies on the robustness of the synchronization within the transmitter and receiver pair [2], [4]-[10]. However, if the communication channel is imperfect and /or there is internal noise at the electronic circuitry the distorted signal at the receiver input might cause considerable synchronization mismatch between the transmitter-receiver pair [11]-[15].

In this paper, we consider the dynamical system first presented in [3] and we investigate the synchronization of the system under noisy channel conditions as well as the case where different noise levels are added in the transmitter and the receiver (internal noise) due to electronics. Moreover, since in every practical implementation of a communication system, the transmitter and receiver electronic elements may be slightly different or operate under slightly different conditions, it is essential to consider the case of the mismatch between the parameters of the transmitter and receiver. The paper is organized as follows: The circuit's description and the synchronization properties are presented in section II. The simulation

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results obtained are shown in section III. Finally, concluding remarks and discussion are given in section IV.

CIRCUIT DESCRIPTION AND SYNCHRONIZATION PROPERTIES

The transmitter and the receiver are identical circuits similar to those in [3]. The circuits include an integrator based 2nd order *RC* resonance loop, a comparator *H* (the circuit's non-linear element), an exclusive OR gate, with an input *M* for the external source of square pulses *M(t)* of period $T=2\pi/\omega$, and a buffer to avoid overloading of the XOR gate. The transmitter-receiver system is shown in Fig. 1. The principle of operation is demonstrated below. Here the chaotic pulses $U^*(t) \propto F(y_1,t)$ drive both the resonance loop of the transmitter and the resonance loop of the receiver.

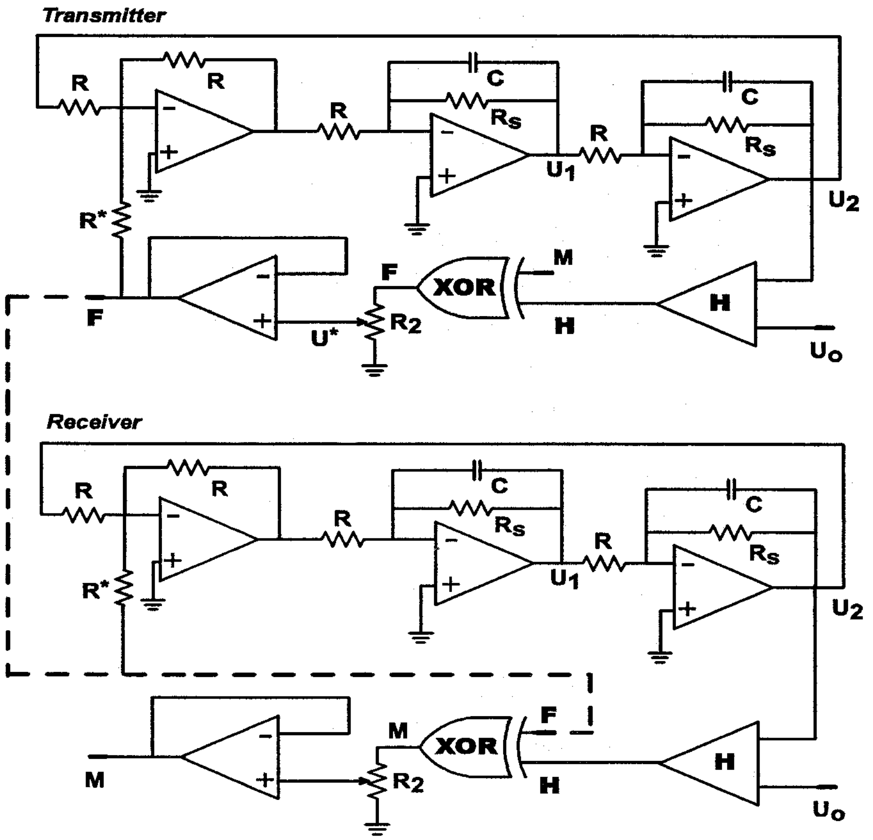

Fig. 1. Schematic diagram of the transmitter-receiver system.

The transmitter-receiver system is governed by the following set of equations:

$$\begin{aligned} \dot{x}_1 &= aF(y_1,t) - bx_1 + y_1 \\ \dot{y}_1 &= -x_1 - by_1 \\ F(y_1,t) &= H(y_1) \oplus M(t) \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_2 &= aF(y_1,t) - bx_2 + y_2 \\ \dot{y}_2 &= -x_2 - by_2 \end{aligned}$$

The subscripts '1' and '2' at the state variables specify the transmitter and the receiver, respectively.

Note the same driving term $F(y_1,t)$ in the equations for the transmitter and the receiver. The following substitutions have been used in the previous system of equations since the parameters are usually written in a dimensionless form:

$$x = \frac{U_1}{U_o},\ y = \frac{U_2}{U_o},\ t = \frac{t}{RC}$$

$$\alpha = \frac{U^* \cdot R}{U_o \cdot R^*},\ b = \frac{R}{R_s} \ (2)$$

$$\omega = \omega_M RC$$

The shifted Heaviside function $H(y)=H(-y-1)$ has the following values $H(-y>1)=1$ and $H(-y\leq 1)=0$, while the symbol $\oplus$ denotes the exclusive-OR operation and $M(t)$ denotes the normalized square pulses of period $2\pi/\omega$ representing the circuit's driving signal.

For zero external drive M to the XOR-gate the circuit exhibits damped oscillations. For all reasonable $(x_o^2 + y_o^2 < 1)$ initial conditions, the corresponding amplitudes of the variables x and y converge exponentially ($\propto e^{-bt}$) to a stable steady state. However, due to the comparator H, for a non-zero drive M the circuit becomes a periodically forced 2nd order non-autonomous non-linear oscillator, exhibiting chaos [1], [2].

Introducing in (1) the error variables $\Delta x = x_2 - x_1$ and, $\Delta y = y_2 - y_1$ we obtain the equations governing the error dynamics:

$$\Delta\dot{x} = b\Delta x + \Delta y\ ,\ \Delta\dot{y} = -\Delta x - b\Delta y \tag{3}$$

The solution of (3) shows the exponential decrease of the errors for all possible initial errors Δx_0 and Δy_0 :

$\Delta x = a\exp(-bt)\cos(t+\phi)$, $\Delta y = a\exp(-bt)\sin(t+\phi)$ (4)

where $a = \sqrt{\Delta x_0^2 + \Delta y_0^2}$ and $\phi = \arctan(\Delta y_0 / \Delta x_0)$

Thus, the synchronization is globally asymptotically stable. This requirement leads to the conclusion that for $\Delta x \to 0$ and $\Delta y \to 0$, the corresponding state variables, are robustly synchronized ($x_2 \to x_1$ and $y_2 \to y_1$). Consequently, the non-linear functions behave in a synchronous way $H(y_2) \to H(y_1)$ as well. This result suggests an extremely simple technique of recovering the signal $M(t)$ at the receiver end. The received signal $F(y_1,t) \propto U^*(t)$ is applied to the XOR unit of the receiver. Due to the sum mod2 property, the signal $M(t)$ can be recovered from the chaotic one $F(y_1,t)$ without any errors, according to:

$$\begin{aligned} &F(y_1,t) \oplus H(y_2) = H(y_1) \oplus M(t) \oplus H(y_2) \to \\ &\to H(y_1) \oplus M(t) \oplus H(y_1) = M(t) \end{aligned} \quad (5)$$

SIMULATION RESULTS

Equations (1) have been numerically integrated. Chaotic oscillations are observed for ω=1.1 and α=2.65. The damping parameter b should not be too large and in the simulation we used b=0.02.

Fig. 2 represents the waveforms for $y_1(t)$ and $y_2(t)$ as well as the difference of the two variables leading to perfect synchronization at about t=250 $(RC)^{-1}$. There is no noise added to the system either internally (due to electronics) or externally (at the channel).

The signal $M_d(t)$ is recovered from the received $F(y_1,t)$ by means of the synchronized $H(y_2)$, (5). In the case examined, the signal $M(t)$ is a train of square periodic pulses (carrying no information), however, $M(t)$ could be a modulated digital signal. Again, the simulation is without adding external or internal noise to the system.

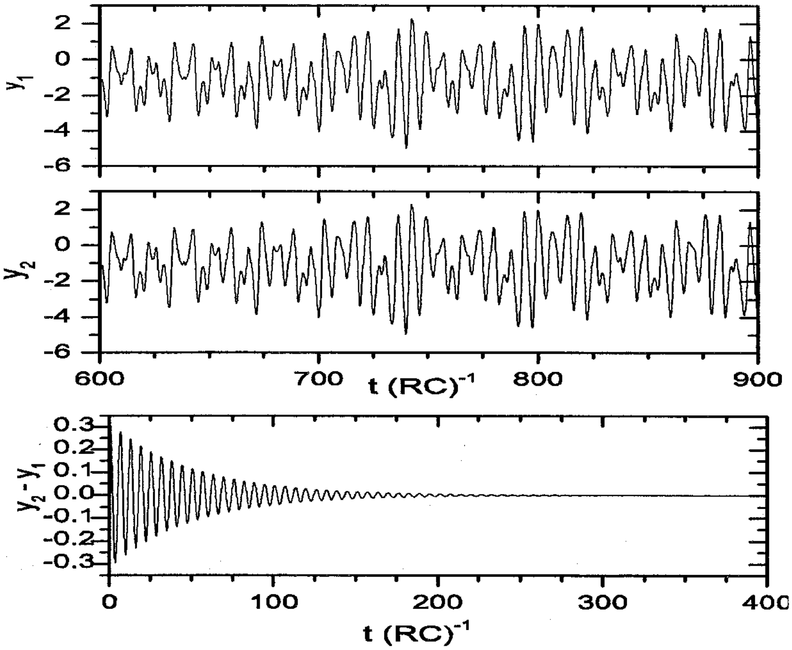

Fig.2. Characteristic waveforms $y_1(t)$, $y_2(t)$ and $(y_2(t)-y_1(t))$.

Fig. 3 and 4 depict the synchronization with the application of external and internal noise respectively. The external noise is applied on the communication channel and in our simulation is represented by white noise added on $F(y_1,t)$, where frequencies greater than RC were cutoff. The internal noise is due to the electronics circuitry and is again applied both on the transmitter and the receiver (added on x_1, y_1 and x_2, y_2 variables). Once again frequencies higher than RC are cutoff.

Different noise amplitudes A, have been utilized ranging from 0.01% to 50% of the mean signal amplitude. As the noise amplitude A is increased, the synchronization of the system continuously deteriorates and is practically destroyed in both cases (external and internal noise) above a certain noise level.

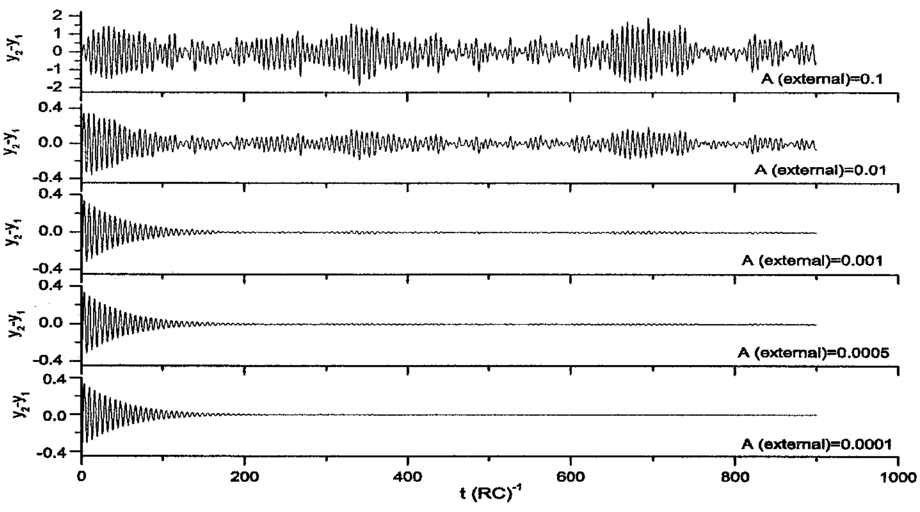

Fig.3. Synchronization with the application of external noise (channel).

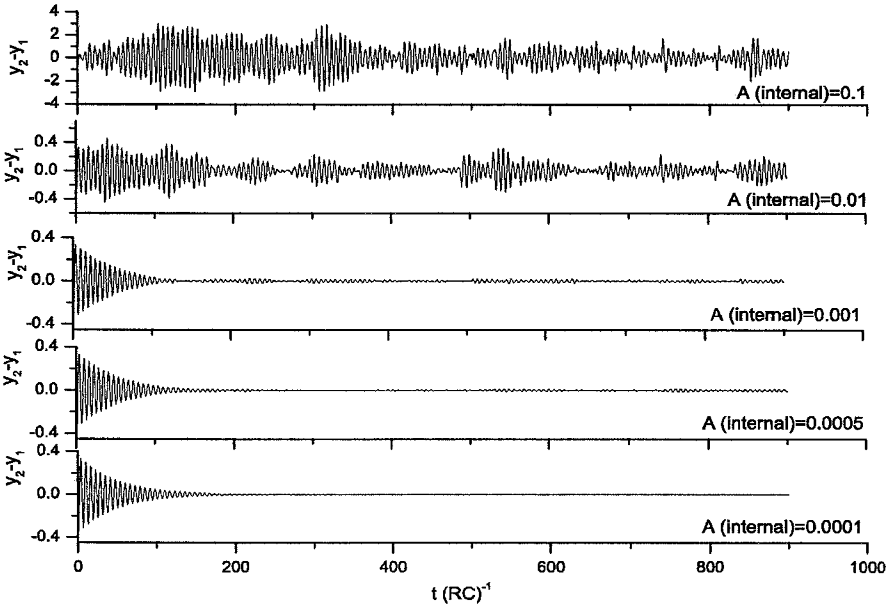

Fig.4. Synchronization with the application of internal noise (electronics).

The artificial noise added at the simulation was produced as follows: A pseudorandom number generator produces an array of random numbers in the interval [0, 1]. The random numbers are equal to the total number of simulation steps. Then the Fourier transform of this series is obtained by standard procedures and amplitudes for frequencies larger than a particular cutoff value are zeroed. The inverse Fourier transform is taken in order to produce the noise series to be used in the subsequent simulation. In our simulation we cutoff frequencies larger than the characteristic frequency of the system RC. By this "noise filtering" procedure we avoid the dependence of the generated noise on the simulation step. Noise was added at every simulation step, to the signal $F(y_1,t)$ coming out of the transmitter (external noise) or to each of the dynamic variables x_1, y_1 and x_2, y_2 of the transmitter and receiver in respect (internal noise). In the latter case, four different noise series were used one for each dynamic variable.

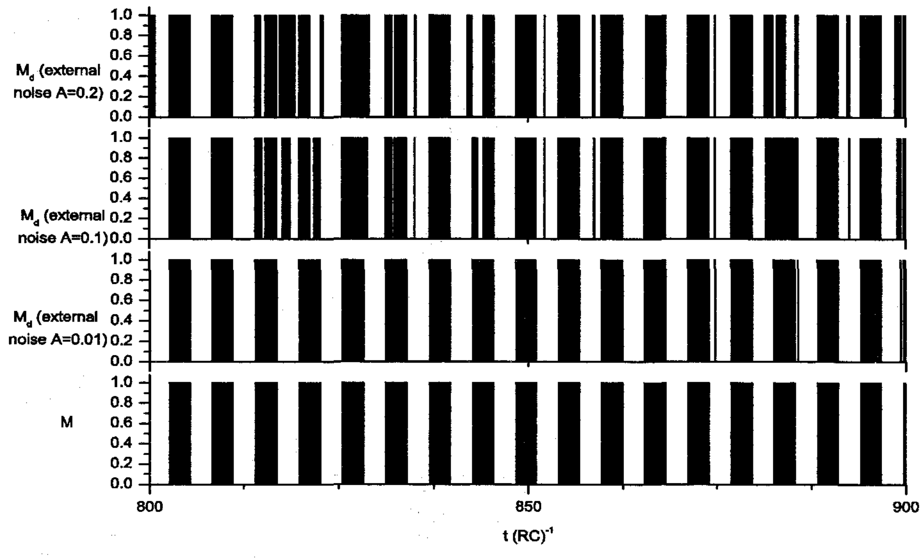

Fig.5. The effect of external noise on the received (decoded) signal $M_d(t)$.

Fig. 5 and 6 represent the sent signal $M(t)$ and the decoded signal $M_d(t)$ (received) for external and internal noise respectively and for different noise amplitudes. It is obvious from the comparison of these two figures that the internal noise damages the signal $M_d(t)$ in a more severe way than the external noise even in the case of lower noise amplitudes.

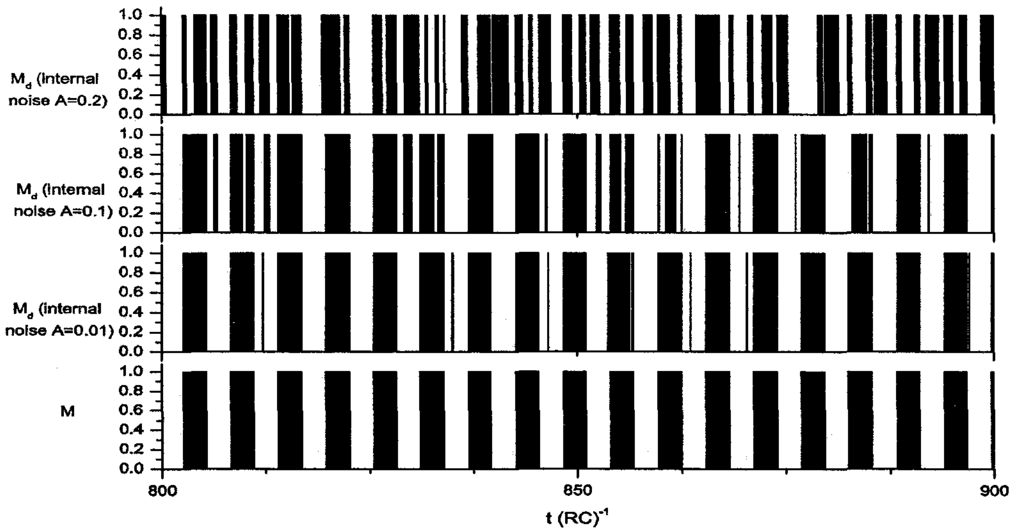

Fig.6. The effect of internal noise on the received (decoded) signal $M_d(t)$.

Moreover, a comparison between the sent and received signals for different amplitudes of external noise shows that the application of a decision circuitry before the signal enters the receiver could drastically improve the decoded signal.

Fig. 7 demonstrates the square mean mismatch $\sqrt{<\Delta y^2>}$ over the initial Δy (=0.3, from Fig.2) and the mean difference $\left[MD = \frac{1}{t}\int_0^t (M(t) - M_d(t))dt \right]$ between signals *M(t)* and $M_d(t)$ in the time interval of the simulation, for external and internal noise respectively. It is apparent that for noise amplitudes greater that 0.3 the mismatch is growing exponentially.

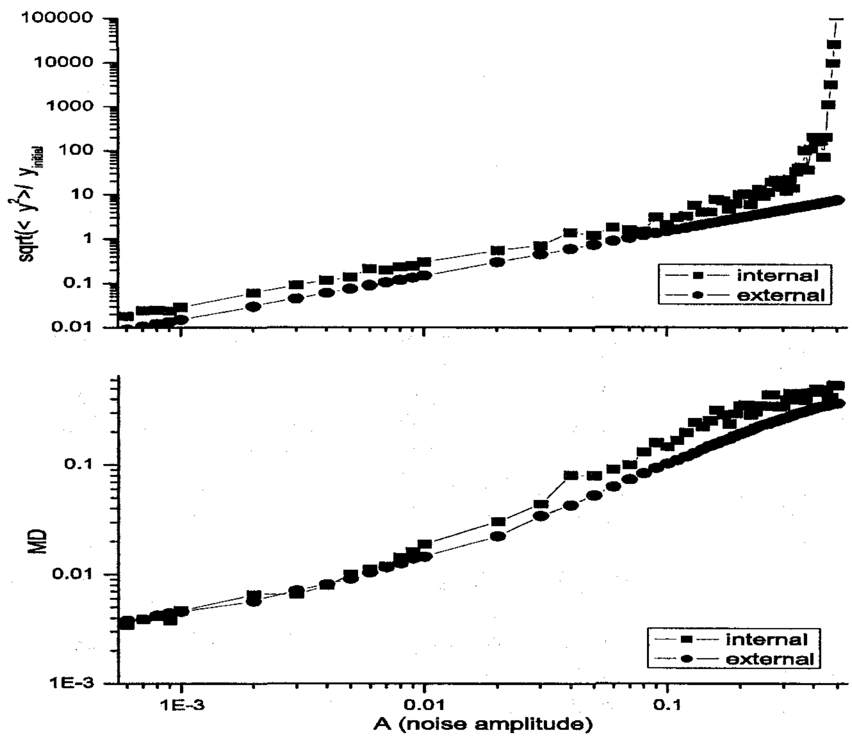

Fig.7. The effect of noise on the square mean mismatch of the y parameters and MD.

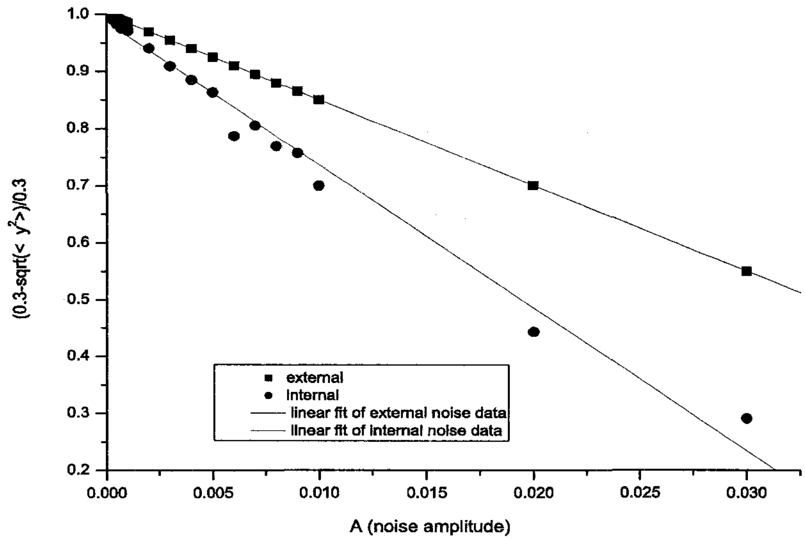

Fig.8. Quality of system' synchronization with noise.

Fig. 8 illustrates the quality of synchronization using a different measure, namely the fractional deviation of the mean square difference of $\sqrt{<\Delta y^2>}$ from the initial Δy (=0.3). This quantity is equal to unity for perfect synchronization. The internal noise, is affecting the system's synchronization more than external noise of the same amplitude level.

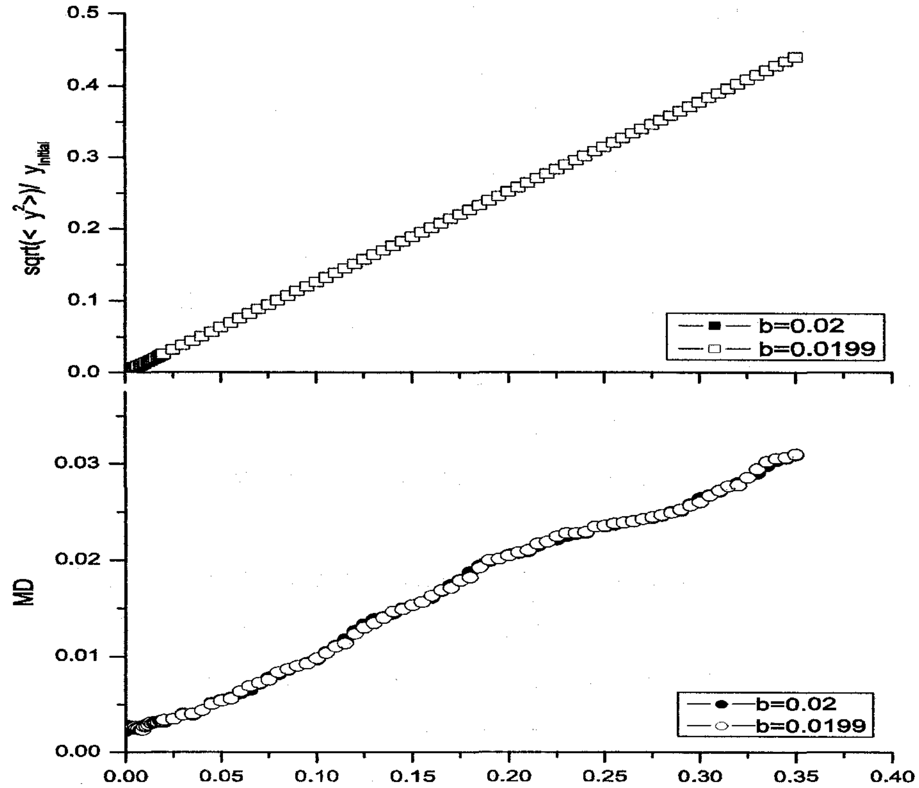

Fig.9. Sensitivity of the system to parameters α and b.

Finally, Fig. 9 illustrates the sensitivity of the system to the parameters α and b. These parameters depend on resistors used in both circuits and naturally may take different values between transmitter and receiver. For α=2.65 and b=0.02 at the transmitter circuit, using a slightly different value of b (b=0.0199) at the receiver circuit does not have any effect on the *MD* or the square mean mismatch of the y parameters regardless of the difference between α parameters of the transmitter and the receiver. The system is also robust in terms of deviations ($\Delta\alpha$) of the values of the parameter α between the transmitter and receiver; a deviation in the order of $\Delta\alpha$=0.35 causes an *MD* of only about 3%.

CONCLUSION

We have studied the influence of the external (channel) and the internal (electronics) noise on the synchronization of the transmitter-receiver pair presented in [3] by numerical simulation of the equations governing the system. The results have shown the robustness of the system although internal noise has more influence than external on the synchronization for the same levels of noise amplitudes. Furthermore, synchronization occurs, even if the parameters of the drive and response system are mismatched, meaning that the system is robust regarding the employed parameters.

Concluding our study, we could state that there is no doubt that the robustness to noise is very advantageous for the synchronization of the system and its realization with off the shelf electronics.

In terms of the security of communication, however, the robustness to the mismatch of the system parameters would be destructive since an enemy in possession of the same device could in principle "tune in" the transmitted signal by adjusting the parameters a and b so that he approximately matches the transmitter's values.

In order to improve security, one would require either a much higher sensitivity of synchronization to system parameters (which would decrease robustness) or a system with a much higher dimensionality in order to increase the number of degrees of freedom an enemy would have to scan in order to synchronize. Implementing systems that would be both robust and also present higher sensitivity could result from a straight forward extension of the very simple system presented in this work.

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